

STATIKA 1

MODUL 7

BANGUNAN PORTAL

Dosen Pengasuh :
Ir. Thamrin Nasution

Materi Pembelajaran :

1. Portal Simetris.

- a) Memikul muatan terpusat tunggal.
- b) Memikul muatan vertikal dan horisontal.
- c) Memikul muatan campuran.

2. Portal Tidak Simetris.

- a) Kolom miring sebelah, memikul muatan terpusat vertikal dan horisontal.
- b) Kolom tinggi sebelah, memikul muatan terpusat vertikal dan horisontal.
- c) Kolom tinggi sebelah, balok overhang, memikul muatan terbagi rata, terpusat vertikal dan horisontal.

WORKSHOP/PELATIHAN

Tujuan Pembelajaran :

- *Mahasiswa memahami dan mengetahui tentang gaya-gaya dalam dari struktur portal simetris, struktur portal tidak simetris, struktur portal dengan overhang, dengan berbagai beban.*

DAFTAR PUSTAKA

- a) Soemono, Ir., "*STATIKA I*", Edisi kedua, Cetakan ke-4, Penerbit ITB, Bandung, 1985.

UCAPAN TERIMA KASIH

Penulis mengucapkan terima kasih yang sebesar-besarnya kepada pemilik hak cipta photo-photo, buku-buku rujukan dan artikel, yang terlampir dalam modul pembelajaran ini.

Semoga modul pembelajaran ini bermanfaat.

Wassalam

Penulis

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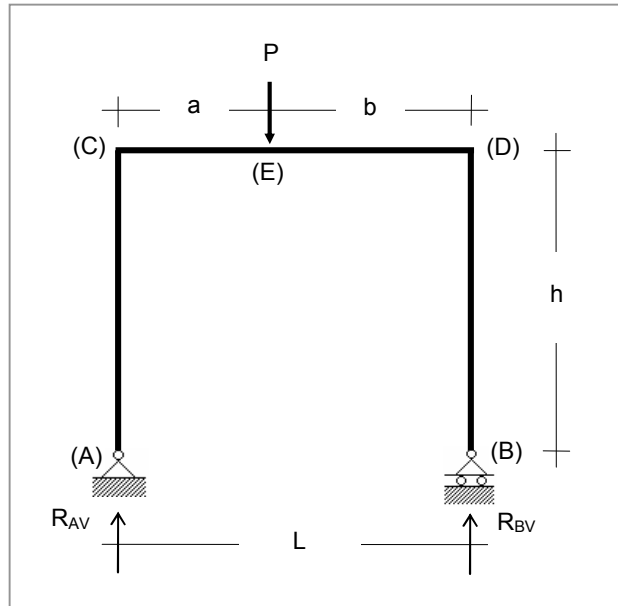
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BANGUNAN PORTAL

Bangunan portal banyak dijumpai sebagai konstruksi bangunan gudang, hangar dan jembatan. Prinsip perhitungan tetap sama dengan balok diatas dua perletakan selagi masih dalam bentuk statis tertentu, yaitu $\Sigma V = 0$, $\Sigma H = 0$ dan $\Sigma M = 0$.

1. PORTAL SIMETRIS.

a). Memikul muatan terpusat tunggal.



Gambar 1 : Bangunan portal simetris, memikul beban terpusat P

Penyelesaian :

a. Reaksi Perletakan.

$$\begin{aligned}\Sigma M_B &= 0, \\ R_{AV} \cdot L - P \cdot b &= 0 \\ R_{AV} &= P \cdot b/L \text{ (ton)}.\end{aligned}$$

$$\begin{aligned}\Sigma M_A &= 0, \\ -R_{BV} \cdot L + P \cdot a &= 0 \\ R_{BV} &= P \cdot a/L \text{ (ton)}.\end{aligned}$$

Kontrol :

$$\begin{aligned}\Sigma V &= 0, \\ R_{AV} + R_{BV} - P &= 0\end{aligned}$$

b. Gaya lintang.

$$\begin{aligned}D_{A-C} &= 0 \\ D_{C-E} &= + R_{AV} \text{ (ton)}. \\ D_{E-D} &= + R_{AV} - P \text{ (ton)}. \\ D_{D-B} &= 0\end{aligned}$$

c. Momen .

$$M_A = 0$$

$$M_C = 0$$

$$M_E = R_{AV} \cdot a = P \cdot a \cdot b / L$$

$$M_D = R_{AV} \cdot L - P \cdot b = + P \cdot b / L \cdot L - P \cdot b = 0$$

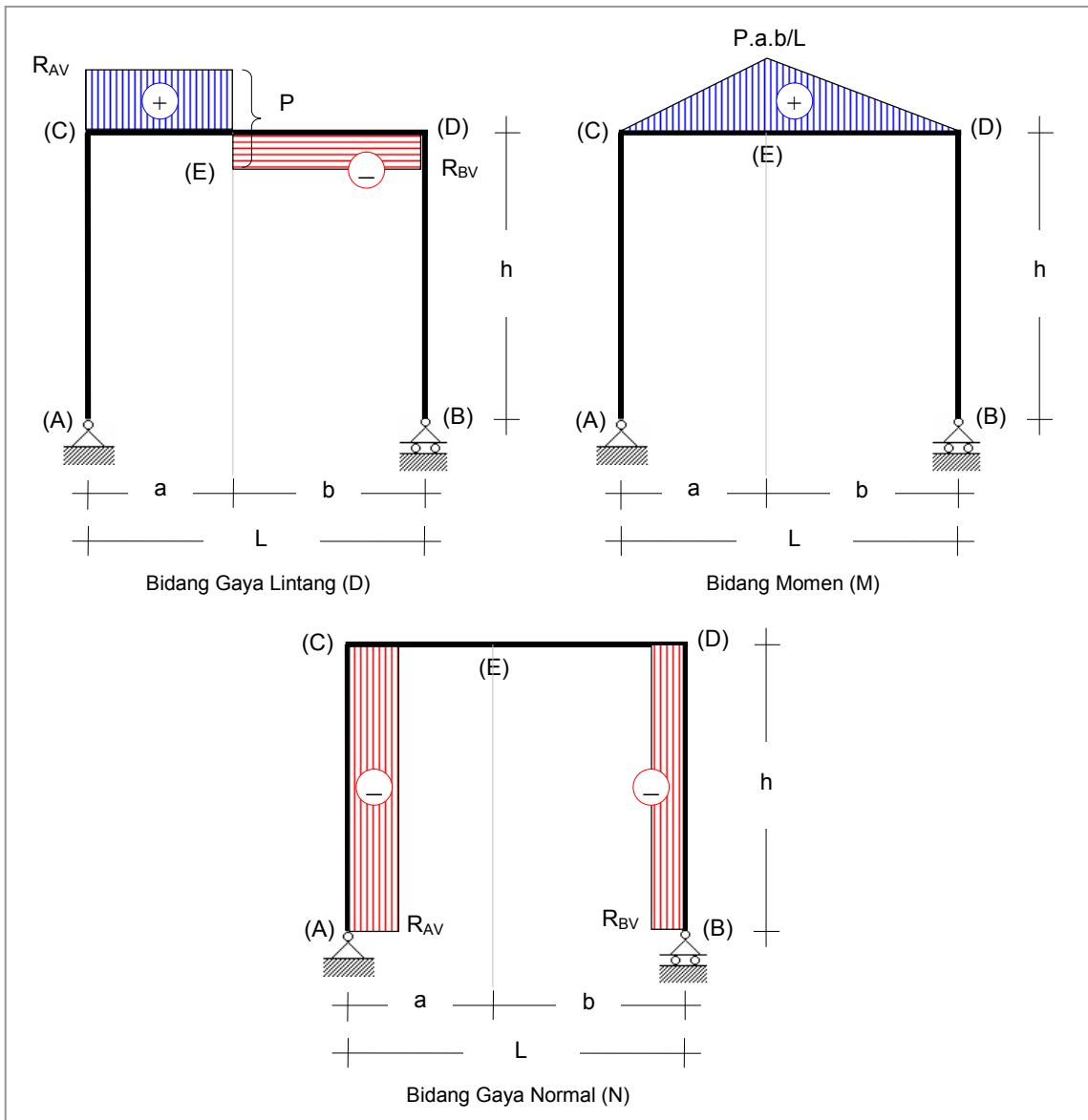
$$M_B = 0$$

d. Gaya Normal.

$$N_{A-C} = - R_{AV} \text{ (ton).}$$

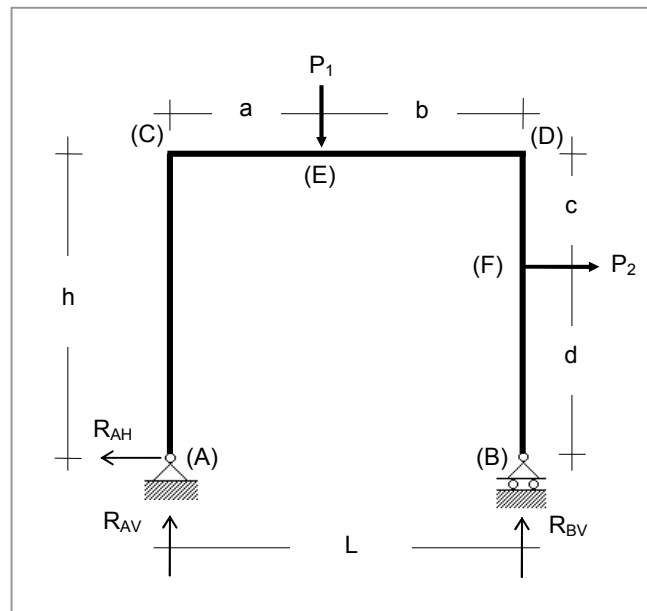
$$N_{C-D} = 0$$

$$N_{B-D} = - R_{BV} \text{ (ton).}$$



Gambar 2 : Bidang gaya lintang, momen dan gaya normal.

b). **Memikul muatan terpusat vertikal dan horizontal.**



Gambar 3 : Bangunan portal simetris, memikul beban terpusat vertikal P_1 dan beban horizontal P_2 .

Penyelesaian :

a. Reaksi Perletakan.

$$\begin{aligned} \sum M_B &= 0, \\ R_{AV} \cdot L - R_{AH} \cdot 0 - P_1 \cdot b + P_2 \cdot d &= 0 \\ R_{AV} &= P_1 \cdot b/L - P_2 \cdot d/L \text{ (ton)}. \\ \sum M_A &= 0, \\ -R_{BV} \cdot L + P_1 \cdot a + P_2 \cdot d &= 0 \\ R_{BV} &= P_1 \cdot a/L + P_2 \cdot d/L \text{ (ton)}. \\ \sum H &= 0, \\ -R_{AH} + P_2 &= 0 \\ R_{AH} &= P_2 \text{ (ton, kekiri)}. \end{aligned}$$

Kontrol :

$$\begin{aligned} \sum V &= 0, \\ R_{AV} + R_{BV} - P_1 &= 0 \end{aligned}$$

b. Gaya lintang.

$$\begin{aligned} D_{A-C} &= + R_{AH} \text{ (ton)}. \\ D_{C-E} &= + R_{AV} \text{ (ton)}. \\ D_{E-D} &= + R_{AV} - P_1 \text{ (ton)}. \\ D_{D-F} &= - R_{AH} \text{ (ton)}. \\ D_{F-B} &= - R_{AH} + P_2 = 0 \text{ (ton)}. \end{aligned}$$

c. Momen.

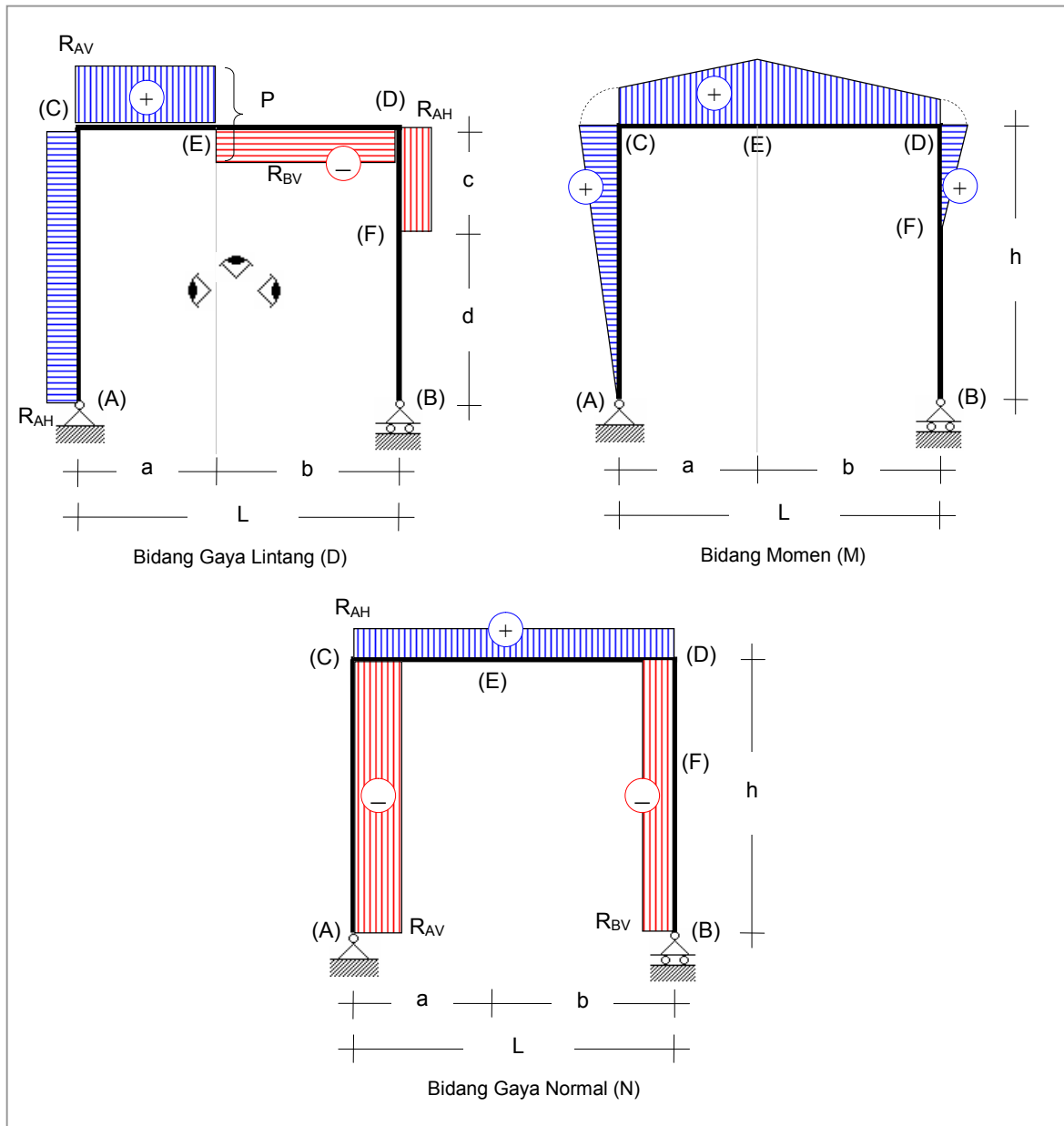
$$\begin{aligned} M_A &= 0 \\ M_C &= + R_{AH} \cdot h \text{ (t.m')}. \\ M_E &= R_{AV} \cdot a + R_{AH} \cdot h \text{ (t.m')}. \\ M_D &= R_{AV} \cdot L + R_{AH} \cdot h - P_1 \cdot b \text{ (t.m')}. \\ M_F &= R_{AV} \cdot L + R_{AH} \cdot d - P_1 \cdot b \text{ (t.m')} = 0. \\ M_B &= 0 \end{aligned}$$

d. Gaya Normal.

$$N_{A-C} = -R_{AV} \text{ ton (tekan, kalau reaksi } R_{AV} \text{ keatas).}$$

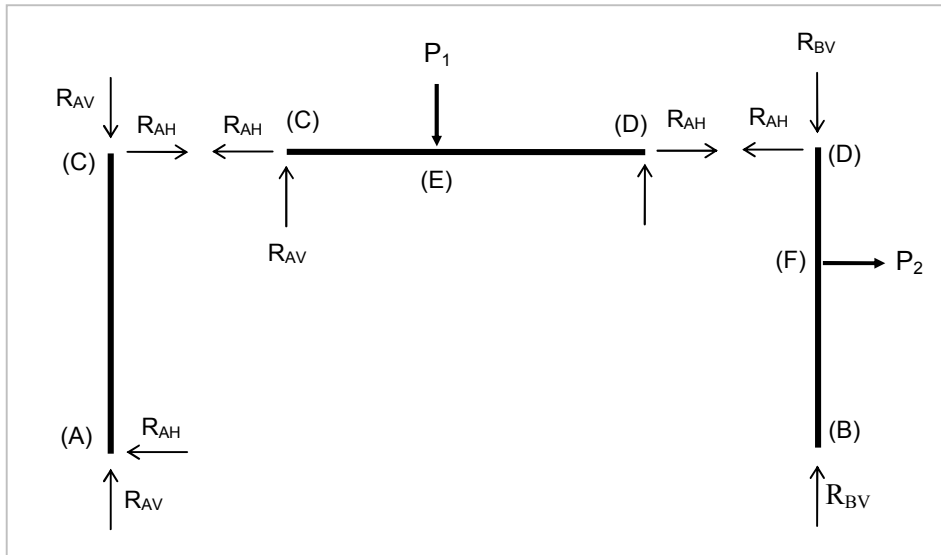
$$N_{C-D} = +R_{AH} \text{ ton (tarik).}$$

$$N_{B-D} = -R_{BV} \text{ ton (tekan).}$$



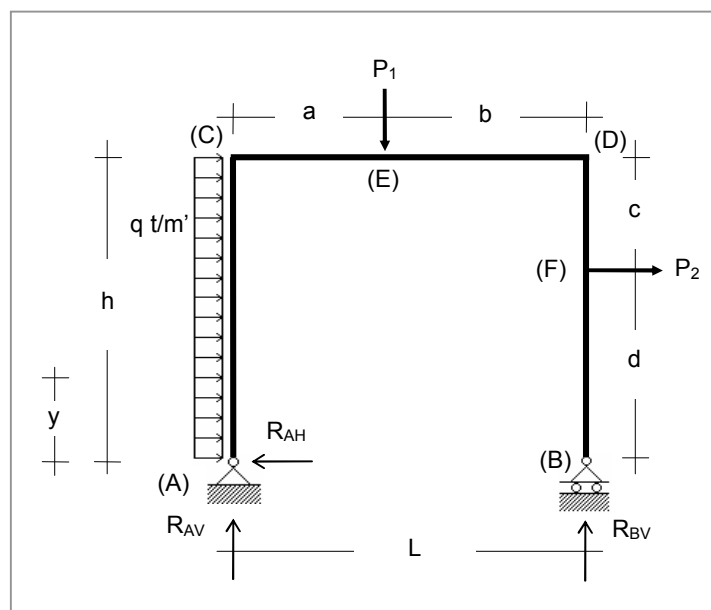
Gambar 4 : Bidang gaya lintang (D), momen (M) dan gaya normal (N).

◊ = mata, cara penglihatan, untuk menggambarkan gaya-gaya dalam dan menetapkan tanda negatif/positif.



Gambar 5 : Distribusi gaya-gaya pada elemen struktur (balok/kolom) dengan cara freebody.

c). **Memikul muatan campuran.**



Gambar 6 : Bangunan portal simetris, memikul beban terpusat vertikal P_1 , beban horisontal P_2 dan $q \text{ t/m}^2$.

Diketahui : Konstruksi seperti tergambar.

$$L = 10 \text{ m}, h = 5 \text{ m}, a = 4 \text{ m}, b = 6 \text{ m}, c = 2 \text{ m}, d = 3 \text{ m}.$$

$$P_1 = 5 \text{ ton}, P_2 = 2 \text{ ton}, q = 1 \text{ t/m}^2.$$

Diminta : Hitung dan gambarkan M, D dan N pada seluruh bentang.

Penyelesaian :

a. Reaksi Perletakan.

$$\Sigma M_B = 0,$$

$$R_{AV} \cdot L - R_{AH} \cdot 0 + q \cdot h \cdot \frac{1}{2} h - P_1 \cdot b + P_2 \cdot d = 0$$

$$R_{AV} = \frac{P_1 \cdot b}{L} - \frac{P_2 \cdot d}{L} - \frac{1}{2} q \cdot \frac{h^2}{L} = \frac{5 \cdot 6}{10} - \frac{2 \cdot 3}{10} - \frac{1}{2} \cdot 1 \cdot \frac{5^2}{10}$$

$$= 3 - 0,6 - 1,25$$

$$R_{AV} = + 1,15 \text{ ton (keatas)}.$$

$$\begin{aligned}\Sigma M_A &= 0, \\ -R_{BV} \cdot L + P_1 \cdot a + P_2 \cdot d + q \cdot h \cdot \frac{1}{2} h &= 0 \\ R_{BV} &= P_1 \cdot a/L + P_2 \cdot d/L + \frac{1}{2} q \cdot h^2/L = 5 \cdot 4/10 + 2 \cdot 3/10 + \frac{1}{2} \cdot 1 \cdot 5^2/10 \\ &= 2 + 0,6 + 1,25 \\ R_{BV} &= + 3,85 \text{ ton (keatas)}.\end{aligned}$$

$$\begin{aligned}\Sigma H &= 0, \\ -R_{AH} + q \cdot h + P_2 &= 0 \\ R_{AH} &= P_2 + q \cdot h = 2 + 1 \cdot 5 \\ R_{AH} &= + 7 \text{ ton (kekiri)}.\end{aligned}$$

Kontrol :

$$\begin{aligned}\Sigma V &= 0, \\ R_{AV} + R_{BV} - P_1 &= 0 \\ 1,15 + 3,85 - 5 &= 0 \quad (\text{memenuhi}).\end{aligned}$$

b. Gaya lintang.

$$\begin{aligned}D_{AC} &= + R_{AH} = + 7 \text{ (ton)}. \\ D_y &= + R_{AH} - q \cdot y \\ D_{y=5m} &= D_{CA} = + R_{AH} - q \cdot h = + 7 - 1 \cdot 5 = + 2 \text{ (ton)}. \\ D_{C-E} &= + R_{AV} = + 1,15 \text{ (ton)}. \\ D_{E-D} &= + R_{AV} - P_1 = 1,15 - 5 = - 3,85 \text{ (ton)}. \\ D_{D-F} &= - R_{AH} + q \cdot h = - 7 + 1 \cdot 5 = - 2 \text{ (ton)}, \text{ atau} \\ D_{D-F} &= - P_2 = - 2 \text{ (ton)}\end{aligned}$$

c. Momen.

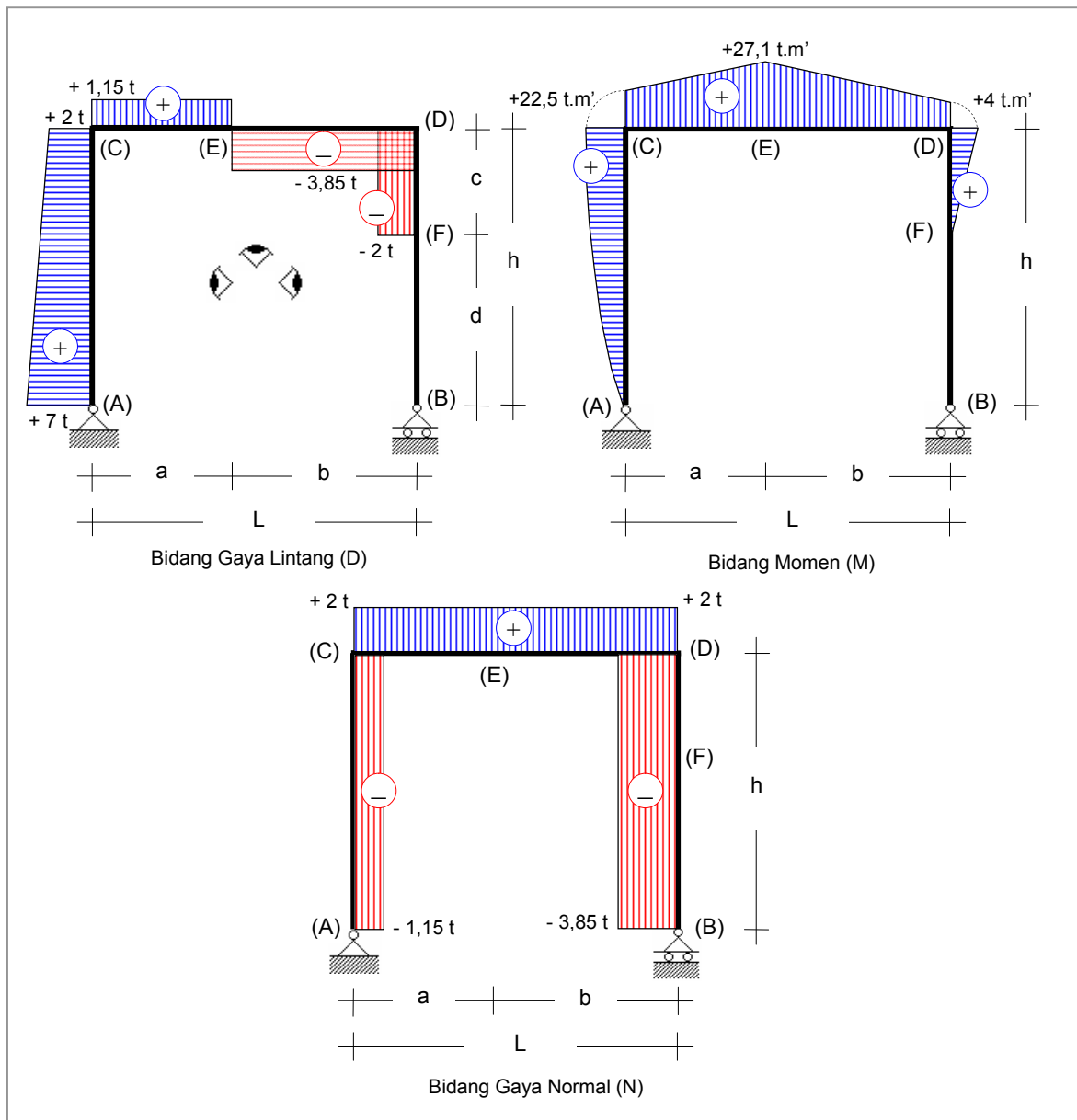
$$\begin{aligned}M_A &= 0 \\ M_y &= + R_{AH} \cdot y - \frac{1}{2} \cdot q \cdot y^2 \\ M_C &= + R_{AH} \cdot h - \frac{1}{2} q \cdot h^2 = 7 \cdot 5 - \frac{1}{2} \cdot 1 \cdot 5^2 = 35 - 12,5 = + 22,5 \text{ (t.m')}. \\ M_E &= + R_{AH} \cdot h - \frac{1}{2} q \cdot h^2 + R_{AV} \cdot a = + 7 \cdot 5 - \frac{1}{2} \cdot 1 \cdot 5^2 + 1,15 \cdot 4 \\ &= 35 - 12,5 + 4,6 \\ M_E &= + 27,1 \text{ (t.m')}. \\ M_D &= + R_{AH} \cdot h - \frac{1}{2} q \cdot h^2 + R_{AV} \cdot L - P_1 \cdot b \\ &= + 7 \cdot 5 - \frac{1}{2} \cdot 1 \cdot 5^2 + 1,15 \cdot 10 - 5 \cdot 6 = 35 - 12,5 + 11,5 - 30 \\ M_D &= + 4 \text{ (t.m')}. \end{aligned}$$

Atau,

$$\begin{aligned}M_D &= + P_2 \cdot c = + 2 \cdot 2 = + 4 \text{ (t.m')}. \\ M_F &= + R_{AH} \cdot d - q \cdot h \cdot (d - \frac{1}{2} h) + R_{AV} \cdot L - P_1 \cdot b \\ &= + 7 \cdot 3 - 1 \cdot 5 \cdot (3 - \frac{1}{2} \cdot 5) + 1,15 \cdot 10 - 5 \cdot 6 \\ &= 21 - 2,5 + 11,5 - 30 \\ M_F &= 0 \\ M_B &= 0\end{aligned}$$

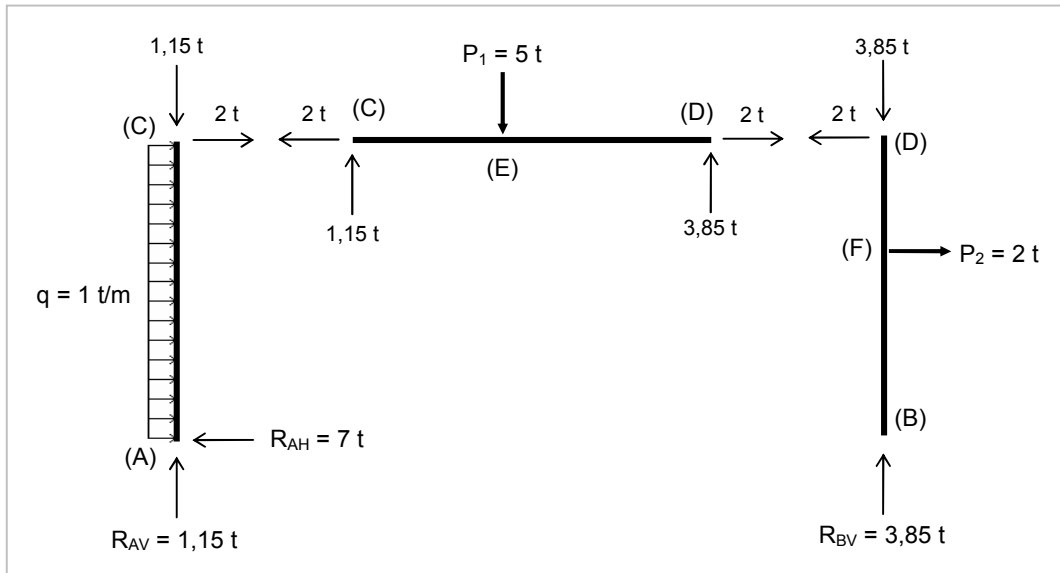
d. Gaya Normal.

$$\begin{aligned}N_{A-C} &= - R_{AV} = - 1,15 \text{ ton (tekan)}. \\ N_{C-D} &= + R_{AH} - q \cdot h = 7 - 1 \cdot 5 = + 2 \text{ ton (tarik)}. \\ N_{B-D} &= - R_{BV} = - 3,85 \text{ ton (tekan)}.\end{aligned}$$



Gambar 7 : Bidang gaya lintang (D), momen (M) dan gaya normal (N).

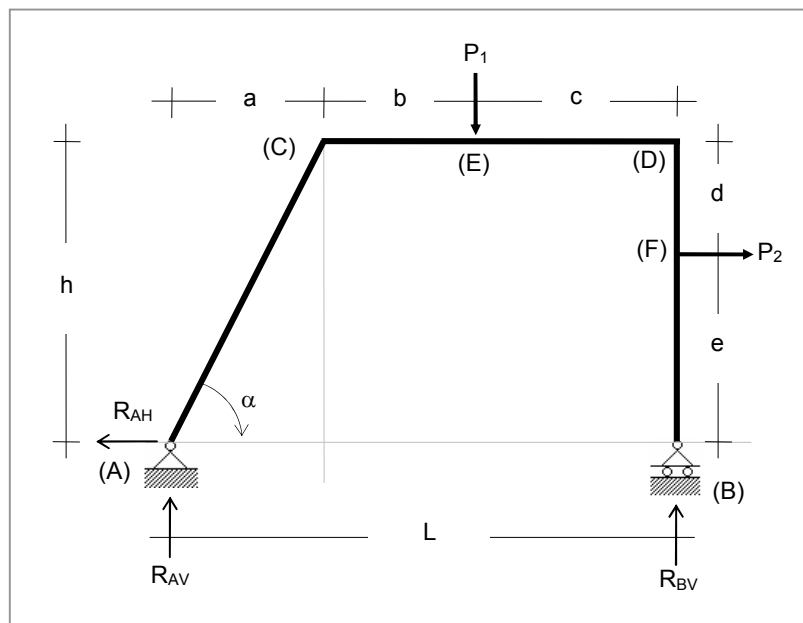
◀ = mata, cara penglihatan, untuk menggambarkan gaya-gaya dalam dan menetapkan tanda negatif/positif.



Gambar 8 : Distribusi gaya-gaya pada elemen struktur (balok/kolom) dengan cara freebody.

2. PORTAL TIDAK SIMETRIS.

a). Kolom miring sebelah, memikul muatan terpusat vertikal dan horisontal.



Gambar 9 : Bangunan portal tidak simetris, memikul beban terpusat vertikal P_1 , beban horisontal P_2 .

Diketahui : Konstruksi seperti tergambar.
 $L = 10 \text{ m}$, $h = 5 \text{ m}$, $a = 2 \text{ m}$, $b = 2 \text{ m}$, $c = 6 \text{ m}$, $d = 2 \text{ m}$, $e = 3 \text{ m}$.
 $P_1 = 5 \text{ ton}$, $P_2 = 2 \text{ ton}$.

Diminta : Hitung dan gambarkan M , D dan N pada seluruh bentang.

Penyelesaian :

a. Reaksi Perletakan.

$$\sum M_B = 0,$$

$$R_{AV} \cdot L - R_{AH} \cdot 0 - P_1 \cdot c + P_2 \cdot e = 0$$

$$R_{AV} = P_1 \cdot c/L - P_2 \cdot e/L = 5 \cdot 6/10 - 2 \cdot 3/10 = 3 - 0,6$$

$$R_{AV} = + 2,4 \text{ ton (keatas).}$$

$$\Sigma M_A = 0,$$

$$- R_{BV} \cdot L + P_1 \cdot (a+b) + P_2 \cdot e = 0$$

$$R_{BV} = P_1 \cdot (a+b)/L + P_2 \cdot e/L = 5 \cdot (2+2)/10 + 2 \cdot 3/10 = 2 + 0,6$$

$$R_{BV} = + 2,6 \text{ ton (keatas).}$$

$$\Sigma H = 0,$$

$$- R_{AH} + P_2 = 0$$

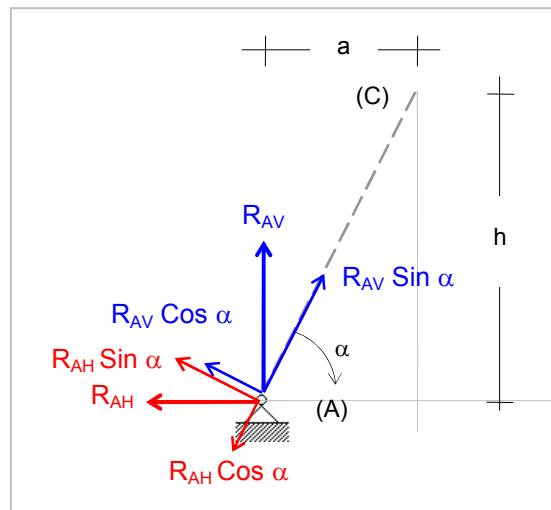
$$R_{AH} = P_2 = + 2 \text{ ton (kekiri).}$$

Kontrol :

$$\Sigma V = 0,$$

$$R_{AV} + R_{BV} - P_1 = 0$$

$$2,4 + 2,6 - 5 = 0 \quad (\text{memenuhi}).$$



Gambar 10 : Komponen reaksi vertikal dan horisontal.

b. Gaya lintang.

$$\alpha = \text{arc tan } (h/a) = \text{arc tan } (5/2)$$

$$= 68^\circ 11' 55''$$

$$D_{A-C} = + R_{AH} \sin \alpha + R_{AV} \cos \alpha = 2 \cdot \sin(68^\circ 11' 55'') + 2,4 \cdot \cos(68^\circ 11' 55'')$$

$$= + 1,857 + 0,891 = + 2,748 \text{ (ton).}$$

$$D_{C-E} = + R_{AV} = + 2,4 \text{ (ton).}$$

$$D_{E-D} = + R_{AV} - P_1 = 2,4 - 5 = - 2,6 \text{ (ton)} = - R_{BV}.$$

$$D_{D-F} = - R_{AH} = - 2 \text{ (ton).}$$

c. Momen.

$$M_A = 0$$

$$M_C = + R_{AH} \cdot h + R_{AV} \cdot a = 2 \cdot 5 + 2,4 \cdot 2 = + 14,8 \text{ (t.m')}.$$

$$M_E = R_{AH} \cdot h + R_{AV} \cdot (a+b) = 2 \cdot 5 + 2,4 \cdot (2+2) = + 19,6 \text{ (t.m')}.$$

$$M_D = R_{AH} \cdot h + R_{AV} \cdot L - P_1 \cdot c = 2 \cdot 5 + 2,4 \cdot 10 - 5 \cdot 6 = + 4 \text{ (t.m')}.$$

Atau,

$$M_D = + P_2 \cdot d = + 2 \cdot 2 = + 4 \text{ (t.m')}.$$

$$M_F = R_{AH} \cdot e + R_{AV} \cdot L - P_1 \cdot c = 2 \cdot 3 + 2,4 \cdot 10 - 5 \cdot 6 = 0 + P^?)$$

$$M_B = 0$$

d. Gaya Normal.

$$\alpha = 68^\circ 11' 55''$$

$$N_{A-C} = + R_{AH} \cos \alpha + R_{AV} \sin \alpha$$

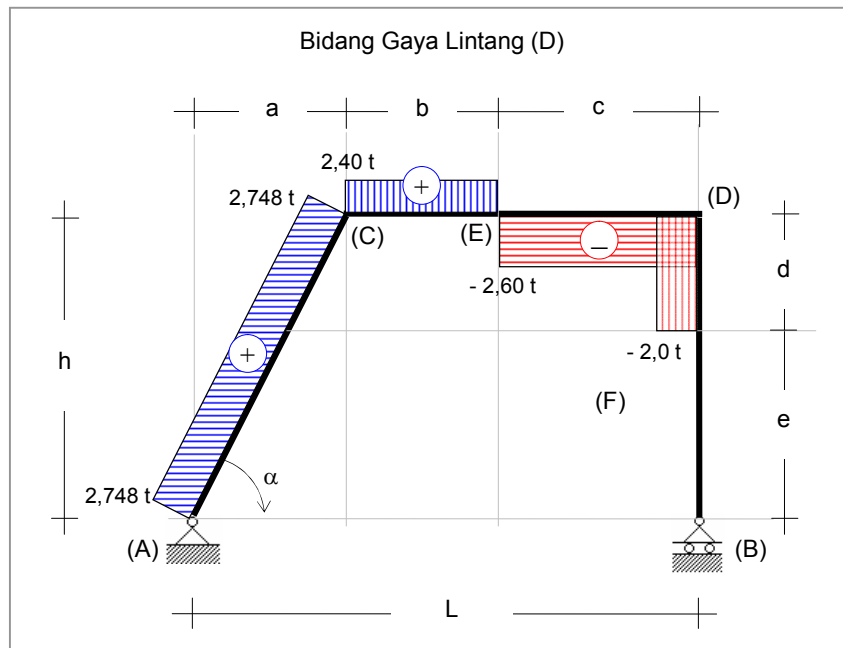
$$= 2 \cdot \cos(68^\circ 11' 55'') - 2,4 \cdot \sin(68^\circ 11' 55'')$$

$$= + 0,743 - 2,228$$

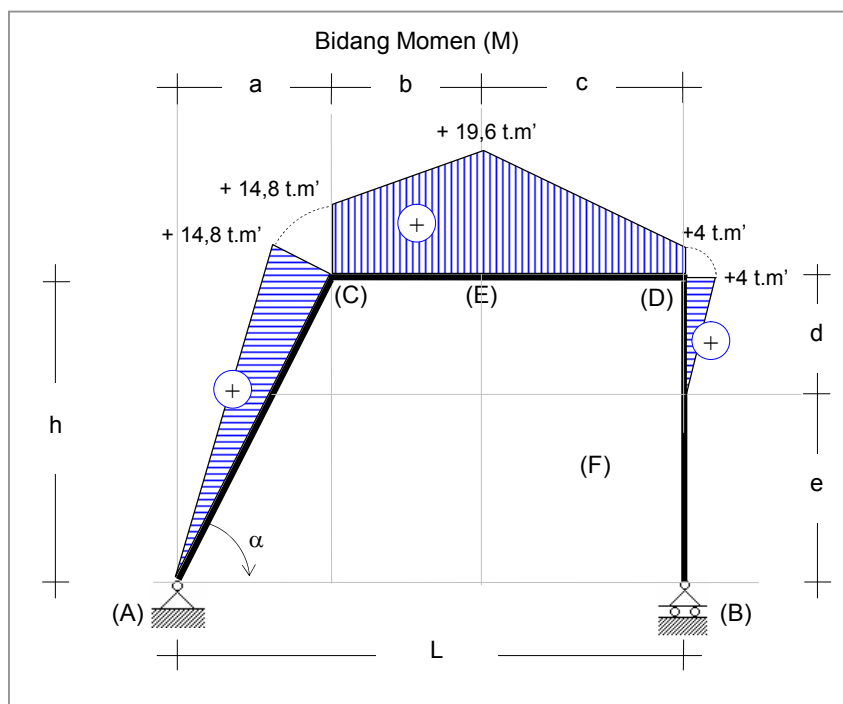
$$= - 1,485 \text{ ton (tekan).}$$

$$N_{C-D} = + R_{AH} = + 2 \text{ ton (tarik).}$$

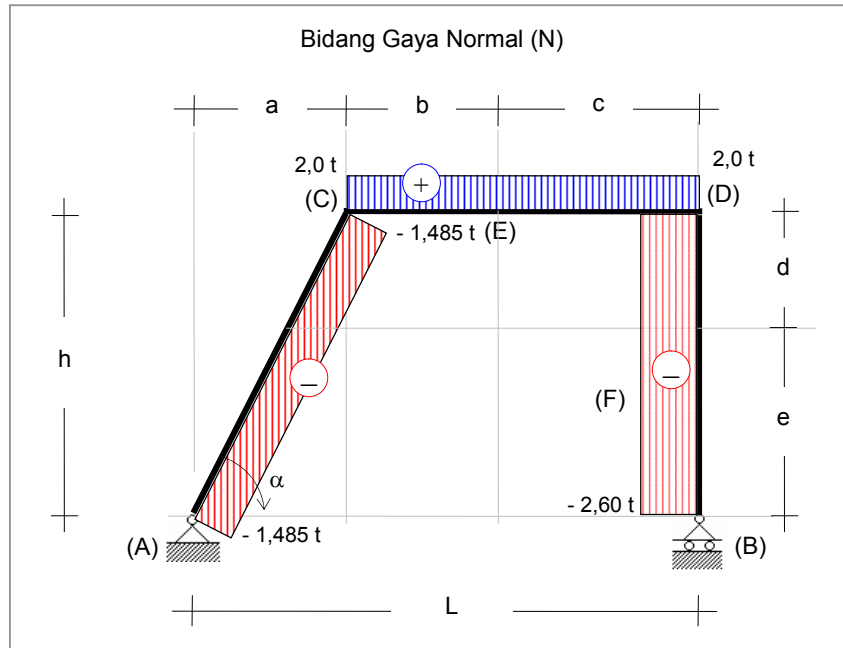
$$N_{B-D} = - R_{BV} = - 2,6 \text{ ton (tekan).}$$



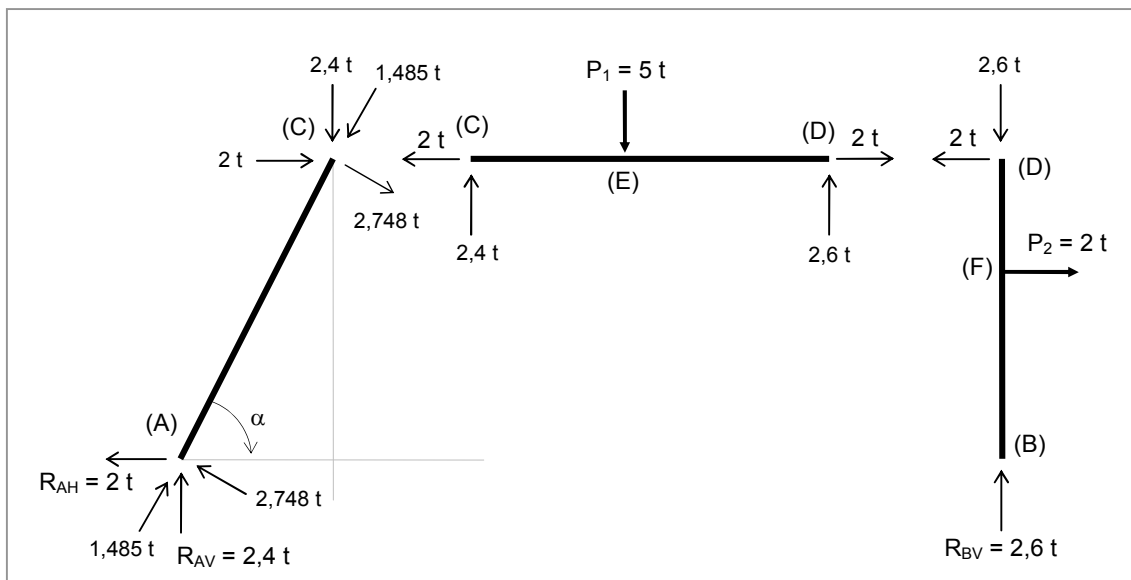
Gambar 11.a. : Bidang gaya lintang.



Gambar 11.b. : Bidang momen.

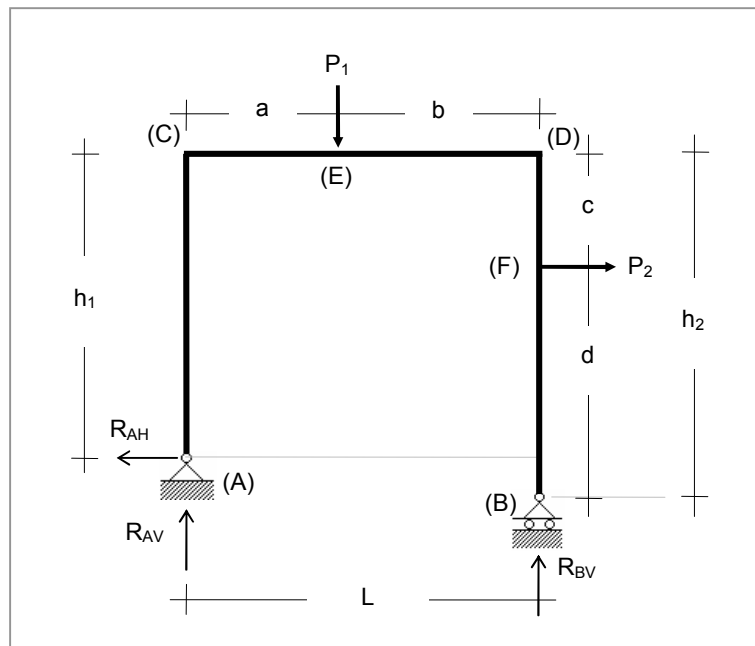


Gambar 11.c. : Bidang gaya normal.



Gambar 12 : Distribusi gaya-gaya pada elemen struktur (balok/kolom) dengan cara freebody.

b). Kolom tinggi sebelah, memikul muatan terpusat vertikal dan horisontal.



Gambar 13 : Portal dengan kaki tinggi sebelah.

Diketahui : Konstruksi seperti tergambar.

$L = 10 \text{ m}$, $h_1 = 5 \text{ m}$, $h_2 = 6 \text{ m}$, $a = 4 \text{ m}$, $b = 6 \text{ m}$, $c = 2 \text{ m}$, $d = 4 \text{ m}$.

$P_1 = 5 \text{ ton}$, $P_2 = 2 \text{ ton}$.

Diminta : Hitung dan gambarkan M, D dan N pada seluruh bentang.

Penyelesaian :

a. Reaksi Perletakan.

$$\sum H = 0,$$

$$-R_{AH} + P_2 = 0$$

$$R_{AH} = +P_2 = +2 \text{ ton (kekiri)}.$$

$$\sum M_B = 0,$$

$$R_{AV} \cdot L - R_{AH} \cdot (h_2 - h_1) - P_1 \cdot b + P_2 \cdot d = 0$$

$$R_{AV} = P_1 \cdot b/L + R_{AH} \cdot (h_2 - h_1)/L - P_2 \cdot d/L \text{ (ton)}.$$

$$= 5 \cdot 6/10 + 2 \cdot (6 - 5)/10 - 2 \cdot 4/10 = 3 + 0,2 - 0,8$$

$$R_{AV} = +2,4 \text{ ton (keatas)}.$$

$$\sum M_A = 0,$$

$$-R_{BV} \cdot L + P_1 \cdot a + P_2 \cdot (h_1 - c) = 0$$

$$R_{BV} = P_1 \cdot a/L + P_2 \cdot (h_1 - c)/L$$

$$= 5 \cdot 4/10 + 2 \cdot (5 - 2)/10 = 2 + 0,6$$

$$R_{BV} = +2,6 \text{ ton (keatas)}.$$

Kontrol :

$$\sum V = 0,$$

$$R_{AV} + R_{BV} - P_1 = 0$$

$$2,4 + 2,6 - 5 = 0 \text{(memenuhi)}.$$

b. Gaya lintang.

$$\begin{aligned}
 D_{A-C} &= + R_{AH} = + 2 \text{ ton.} \\
 D_{C-E} &= + R_{AV} = + 2,4 \text{ ton.} \\
 D_{E-D} &= + R_{AV} - P_1 = + 2,4 - 5 = - 2,6 \text{ ton.} \\
 D_{D-F} &= - R_{AH} = - 2 \text{ ton, atau} \\
 D_{D-F} &= - P_2 = - 2 \text{ ton.}
 \end{aligned}$$

c. M o m e n .

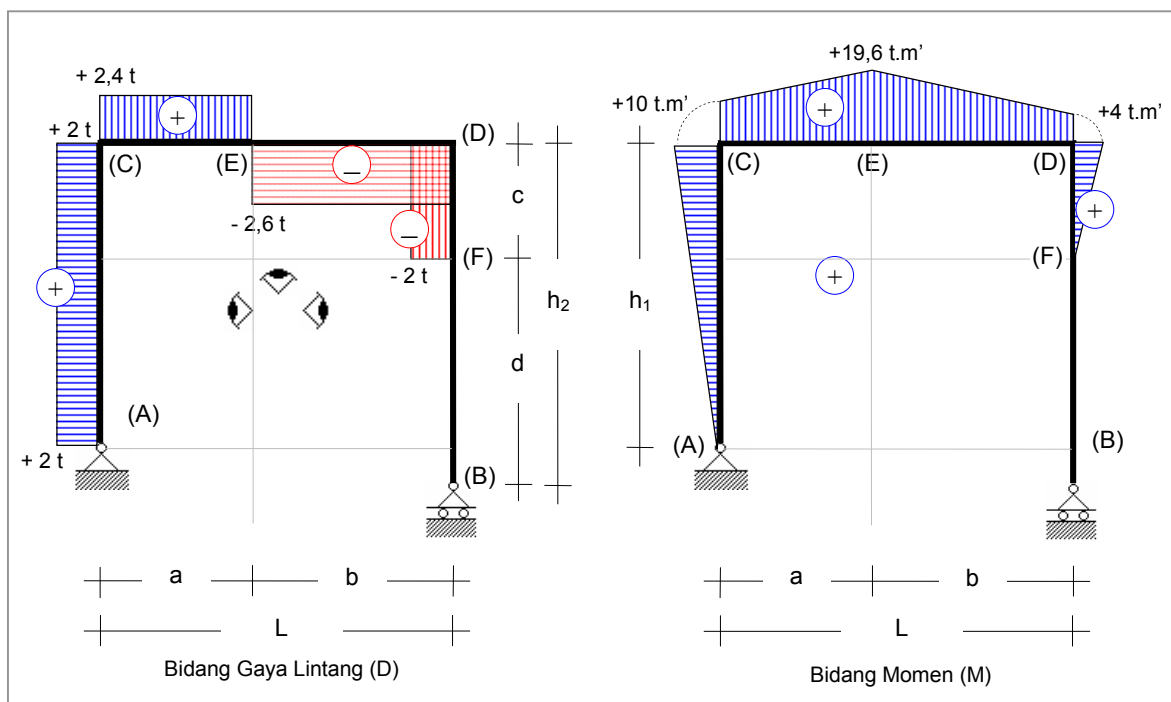
$$\begin{aligned}
 M_A &= 0 \\
 M_C &= + R_{AH} \cdot h = + 2 \cdot 5 = + 10 \text{ t.m}' \\
 M_E &= + R_{AV} \cdot a + R_{AH} \cdot h = + 2,4 \cdot 4 + 2 \cdot 5 = 19,6 \text{ t.m}' \\
 M_D &= + R_{AV} \cdot L + R_{AH} \cdot h - P_1 \cdot b = 2,4 \cdot 10 + 2 \cdot 5 - 5 \cdot 6 \\
 &= 24 + 10 - 30 \\
 M_D &= + 4 \text{ t.m}'
 \end{aligned}$$

Atau,

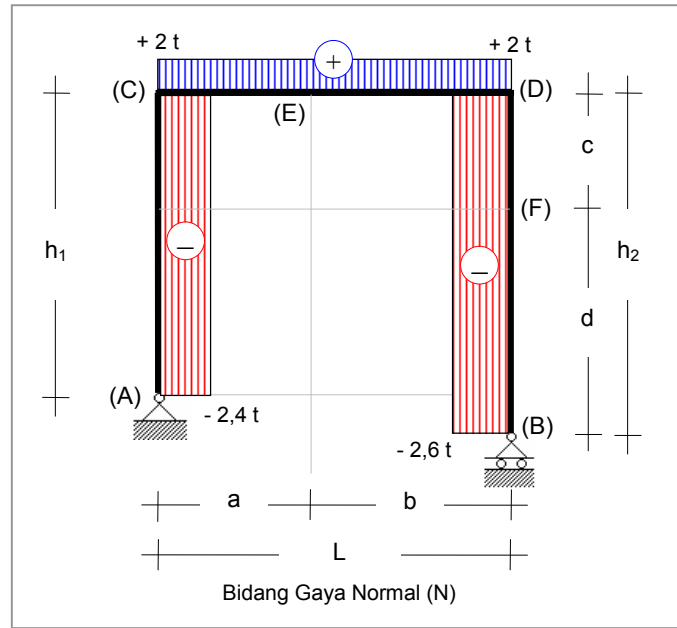
$$\begin{aligned}
 M_D &= + P_2 \cdot c = + 2 \cdot 2 = + 4 \text{ t.m}' \\
 M_F &= + R_{AV} \cdot L + R_{AH} \cdot (h_1 - c) - P_1 \cdot b \text{ (dari kiri)} \\
 &= + 2,4 \cdot 10 + 2 \cdot (5 - 2) - 5 \cdot 6 = 24 + 6 - 30 \\
 M_F &= 0 \\
 M_B &= 0
 \end{aligned}$$

d. Gaya Normal.

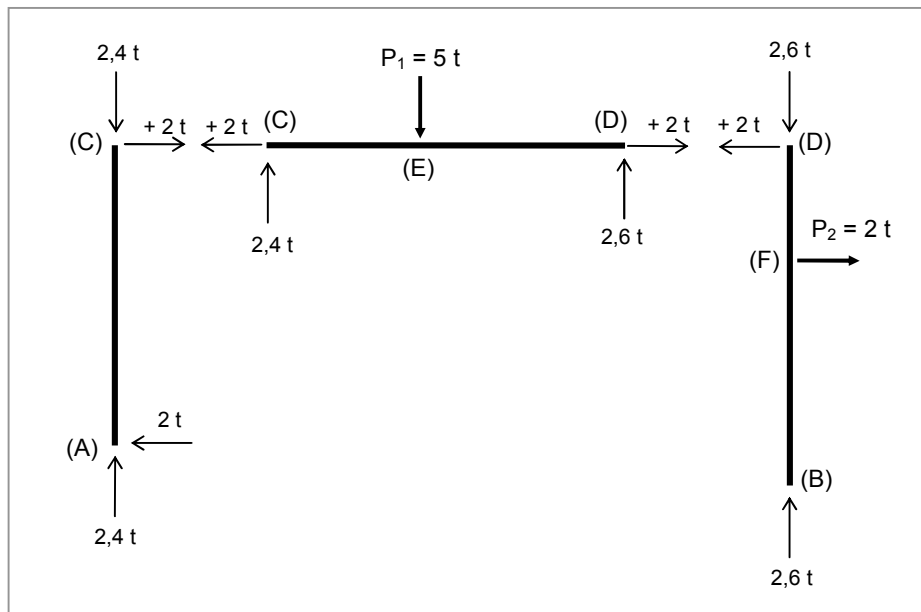
$$\begin{aligned}
 N_{A-C} &= - R_{AV} = - 2,4 \text{ ton (tekan).} \\
 N_{C-D} &= + R_{AH} = + 2 \text{ ton (tarik).} \\
 N_{B-D} &= - R_{BV} = - 2,6 \text{ ton (tekan).}
 \end{aligned}$$



Gambar 14.a : Bidang gaya lintang dan momen.

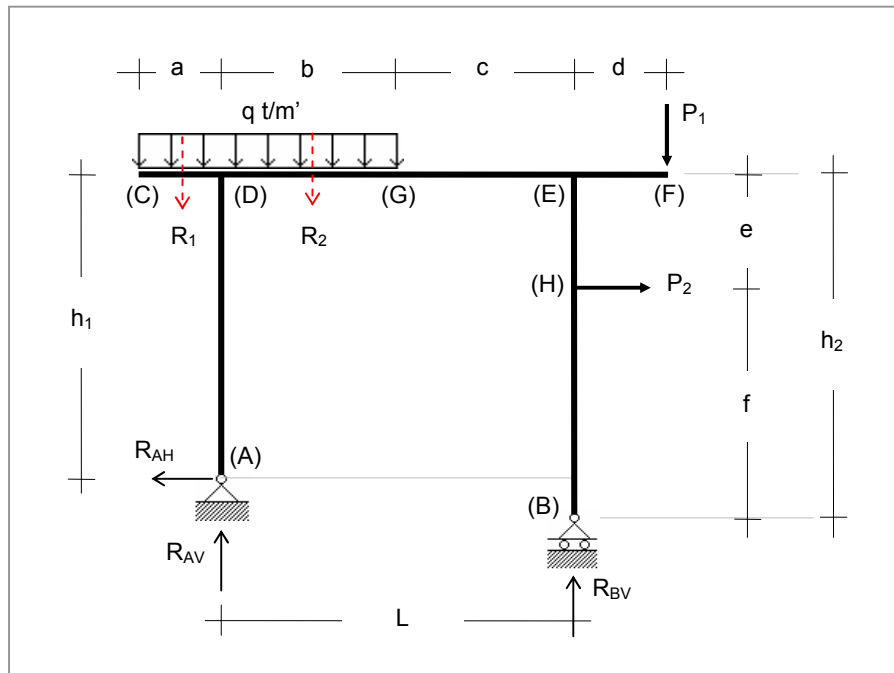


Gambar 14.b : Bidang gaya normal.



Gambar 15 : Distribusi gaya-gaya pada elemen struktur (balok/kolom) dengan cara freebody.

c). Kolom tinggi sebelah, balok overhang, memikul muatan terbagi rata, terpusat vertikal dan horisontal.



Gambar 16 : Portal dengan overhang pada kiri kanan.

Diketahui : Konstruksi seperti tergambar.

$L = 10 \text{ m}$, $h_1 = 5 \text{ m}$, $h_2 = 6 \text{ m}$, $a = 1 \text{ m}$, $b = 4 \text{ m}$, $c = 6 \text{ m}$, $d = 2 \text{ m}$,
 $e = 2 \text{ m}$, $f = 4 \text{ m}$, $q = 1 \text{ t/m'}$, $P_1 = 5 \text{ ton}$, $P_2 = 2 \text{ ton}$.

Diminta : Hitung dan gambarkan M , D dan N pada seluruh bentang.

Penyelesaian :

a. Reaksi Perletakan.

$$R_1 = q \cdot a = 1 \cdot 1 = 1 \text{ ton.}$$

$$R_2 = q \cdot b = 1 \cdot 4 = 4 \text{ ton.}$$

$$\Sigma H = 0,$$

$$-R_{AH} + P_2 = 0$$

$$R_{AH} = +P_2 = 2 \text{ ton (kekiri).}$$

$$\Sigma M_B = 0,$$

$$R_{AV} \cdot L - R_{AH} \cdot (h_2 - h_1) - R_1 \cdot (L + \frac{1}{2}a) - R_2 \cdot (L - \frac{1}{2}b) + P_1 \cdot d + P_2 \cdot f = 0$$

$$R_{AV} = \frac{R_{AH} \cdot (h_2 - h_1) + R_1 \cdot (L + \frac{1}{2}a) + R_2 \cdot (L - \frac{1}{2}b) - P_1 \cdot d - P_2 \cdot f}{L}$$

$$= \frac{2 \cdot (6 - 5) + 1 \cdot (10 + \frac{1}{2} \cdot 1) + 4 \cdot (10 - \frac{1}{2} \cdot 4) - 5 \cdot 2 - 2 \cdot 4}{10}$$

$$= \frac{0,2 + 1,05 + 3,2 - 1 - 0,8}{10}$$

$$R_{AV} = +2,65 \text{ ton (keatas).}$$

$$\Sigma M_A = 0,$$

$$-R_{BV} \cdot L + P_1 \cdot (L + d) + P_2 \cdot (h_1 - e) - R_1 \cdot \frac{1}{2}a + R_2 \cdot \frac{1}{2}b = 0$$

$$R_{BV} = \frac{P_1 \cdot (L + d) + P_2 \cdot (h_1 - e) - R_1 \cdot \frac{1}{2}a + R_2 \cdot \frac{1}{2}b}{L}$$

$$= \frac{5 \cdot (10 + 2) + 2 \cdot (5 - 2) - 1 \cdot \frac{1}{2} \cdot 1 + 4 \cdot \frac{1}{2} \cdot 4}{10}$$

$$= 6 + 0,6 - 0,05 + 0,8$$

$$R_{BV} = + 7,35 \text{ ton (keatas).}$$

Kontrol :

$$\begin{aligned} \Sigma V &= 0, \\ R_{AV} + R_{BV} - R_1 - R_2 - P_1 &= 0 \\ 2,65 + 7,35 - 1 - 4 - 5 &= 0 \dots(\text{memenuhi}). \end{aligned}$$

b. Gaya lintang.

$$\begin{aligned} D_{A-D} &= + R_{AH} = + 2 \text{ ton.} \\ D_{D-A} &= D_{A-D} = + 2 \text{ ton.} \\ D_{DC} &= - q \cdot a = - 1 \cdot 1 = - 1 \text{ ton.} \\ D_{DG} &= + R_{AV} + D_{DC} = + R_{AV} - q \cdot a = + 2,65 - 1 = + 1,65 \text{ ton.} \\ D_{G-E} &= D_{DG} - q \cdot b = + R_{AV} - q \cdot (a + b) = + 2,65 - 1 \cdot (1 + 4) = - 2,35 \text{ ton.} \\ D_{E-G} &= D_{G-E} = - 2,35 \text{ ton.} \\ D_{E-F} &= D_{E-G} + R_{BV} = + R_{AV} - q \cdot (a + b) + R_{BV} = - 2,35 + 7,35 = + 5 \text{ ton.} \end{aligned}$$

Atau,

$$\begin{aligned} D_{E-F} &= + P_1 = + 5 \text{ ton.} \\ D_{E-G} &= - R_{AH} = - 2 \text{ ton.} \end{aligned}$$

Atau,

$$D_{E-G} = - 2 \text{ ton.}$$

c. Momen.

$$\begin{aligned} M_A &= 0 \\ M_{DA} &= + R_{AH} \cdot h = + 2 \cdot 5 = + 10 \text{ t.m}^2. \\ M_{DC} &= - \frac{1}{2} q \cdot a^2 = - \frac{1}{2} \cdot 1 \cdot 1^2 = - 0,50 \text{ t.m}^2. \\ M_{DG} &= + R_{AH} \cdot h - \frac{1}{2} q \cdot a^2 = + 2 \cdot 5 - \frac{1}{2} \cdot 1 \cdot 1 = + 9,50 \text{ t.m}^2. \\ M_G &= + R_{AV} \cdot b + R_{AH} \cdot h - \frac{1}{2} q \cdot (a + b)^2 \\ &= + 2,65 \cdot 4 + 2 \cdot 5 - \frac{1}{2} \cdot 1 \cdot (1+4)^2 = + 10,60 + 10 - 12,50 \\ M_G &= + 8,10 \text{ t.m}^2. \\ M_{EG} &= + R_{AV} \cdot L + R_{AH} \cdot h_1 - R_1 \cdot (L + \frac{1}{2} a) - R_2 \cdot (L - \frac{1}{2} b) \\ &= + 2,65 \cdot 10 + 2 \cdot 5 - 1 \cdot (10 + \frac{1}{2} \cdot 1) - 4 \cdot (10 - \frac{1}{2} \cdot 4) \\ &= + 26,5 + 10 - 10,5 - 32 \\ M_{EG} &= - 6 \text{ t.m}^2. \\ M_{EF} &= - P_1 \cdot d = - 5 \cdot 2 = - 10 \text{ t.m}^2. \\ M_{EB} &= + R_{AV} \cdot L + R_{AH} \cdot h_1 - R_1 \cdot (L + \frac{1}{2} a) - R_2 \cdot (L - \frac{1}{2} b) + P_1 \cdot d \\ &= + 2,65 \cdot 10 + 2 \cdot 5 - 1 \cdot (10 + \frac{1}{2} \cdot 1) - 4 \cdot (10 - \frac{1}{2} \cdot 4) + 5 \cdot 2 \\ &= + 26,5 + 10 - 10,5 - 32 + 10 \\ M_{EB} &= + 4 \text{ t.m}^2. \end{aligned}$$

Atau,

$$\begin{aligned} M_{EB} &= + P_2 \cdot e = + 2 \cdot 2 = + 4 \text{ t.m}^2. \\ M_H &= + R_{AV} \cdot L + R_{AH} \cdot (h_1 - e) - R_1 \cdot (L + \frac{1}{2} a) - R_2 \cdot (L - \frac{1}{2} b) + P_1 \cdot d \\ &= + 2,65 \cdot 10 + 2 \cdot (5 - 2) - 1 \cdot (10 + \frac{1}{2} \cdot 1) - 4 \cdot (10 - \frac{1}{2} \cdot 4) + 5 \cdot 2 \\ &= + 26,5 + 6 - 10,5 - 32 + 10 \\ M_H &= 0 \text{ t.m}^2. \\ M_F &= 0 \text{ t.m}^2 ; \\ M_B &= 0 \text{ t.m}^2. \end{aligned}$$

Momen maksimum positif pada daerah D-E,

$$\begin{aligned} M_{X1} &= R_{AV} \cdot x_1 + R_{AH} \cdot h_1 - R_1 \cdot (a + x_1) - \frac{1}{2} \cdot q \cdot x_1^2 \\ D_{X1} &= dM_{X1}/dx_1 = 0 \\ R_{AV} - R_1 - q \cdot x_1 &= 0 \\ x_1 &= (R_{AV} - R_1)/q \end{aligned}$$

$$= (2,65 - 1)/1$$

$$x_1 = 1,65 \text{ m (dari D)}$$

$$M_{\text{maks}} = 2,65 \cdot 1,65 + 2 \cdot 5 - 1 \cdot (1 + 1,65) - \frac{1}{2} \cdot 1 \cdot (1,65)^2$$

$$= + 10,36125 \text{ t.m'}$$

Letak titik dimana momen sama dengan nol (dari kiri),

$$M_{X_2} = R_{AV} \cdot (b + x_2) + R_{AH} \cdot h_1 - R_1 \cdot (\frac{1}{2} \cdot a + b + x_2) - R_2 \cdot (\frac{1}{2} \cdot b + x_2) = 0$$

$$2,65 \cdot (4 + x_2) + 2 \cdot 5 - 1 \cdot (\frac{1}{2} \cdot 1 + 4 + x_2) - 4 \cdot (\frac{1}{2} \cdot 4 + x_2) = 0$$

$$2,65 x_2 - x_2 - 4 x_2 + 2,65 \cdot 4 + 2 \cdot 5 - 1 \cdot 4,5 - 4 \cdot 2 = 0$$

$$- 2,35 x_2 + 10,6 + 10 - 4,5 - 8 = 0$$

$$x_2 = (20,6 - 12,5)/2,35$$

$$= 3,45 \text{ m (dari G)}$$

Maka letak momen sama dengan nol dari titik D,

$$x_3 = b + x_2 = 4 + 3,45 = 7,45 \text{ m}$$

Apabila letak momen sama dengan nol ini dihitung dari kanan kekiri,

$$M_{X_4} = R_{BV} \cdot x_4 + P_2 \cdot e - P_1 \cdot (d + x_4) = 0$$

$$7,35 \cdot x_4 + 2 \cdot 2 - 5 \cdot (2 + x_4) = 0$$

$$7,35 x_4 - 5 x_4 + 4 - 10 = 0$$

$$2,35 x_4 - 6 = 0$$

$$x_4 = 6/2,35 = 2,55 \text{ m (dari E)}$$

$$(L - x_4) = 10 - 2,55 = 7,45 \text{ m (dari D)} \quad \dots(\text{memenuhi})$$

d. Gaya Normal.

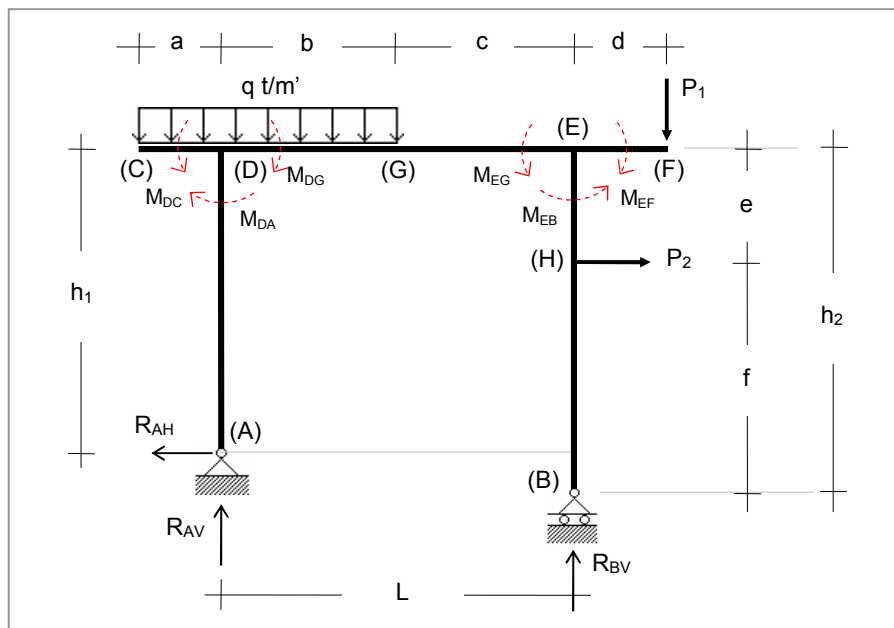
$$N_{A-D} = - R_{AV} = - 2,65 \text{ ton (tekan)}$$

$$N_{C-D} = 0$$

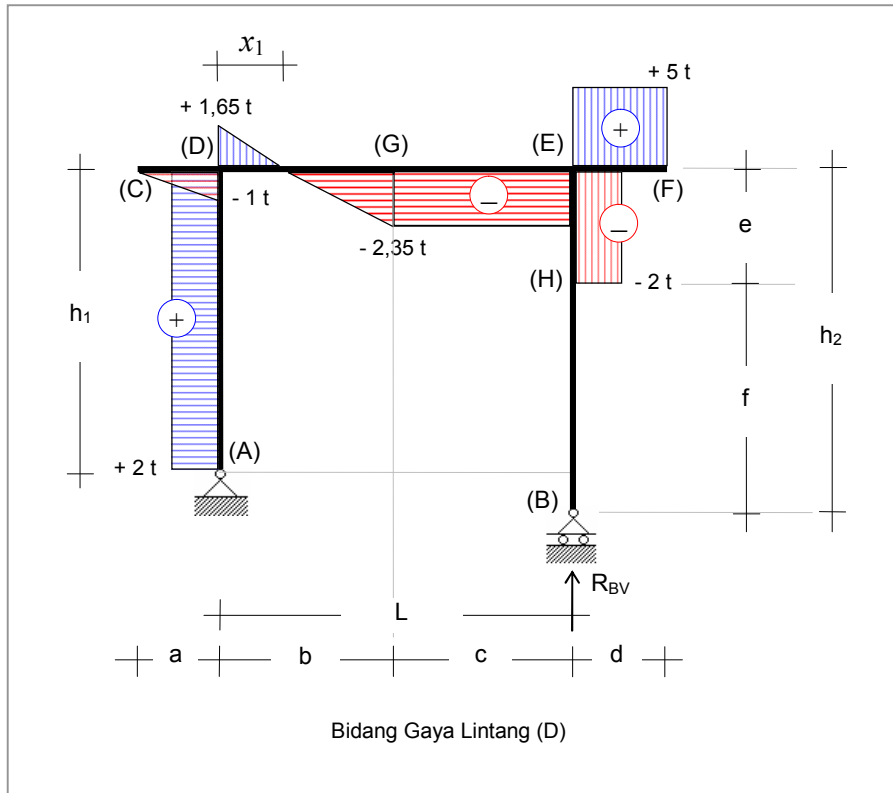
$$N_{D-E} = + R_{AH} = + 2 \text{ ton (tarik)}$$

$$N_{E-B} = - R_{BV} = - 7,35 \text{ ton (tekan)}$$

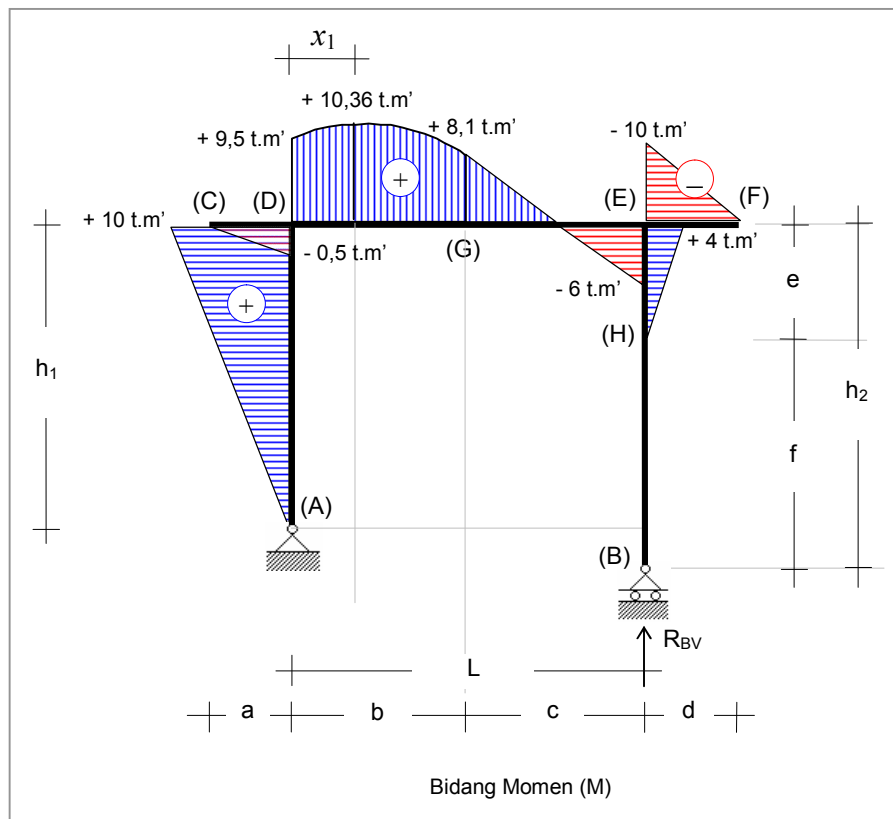
$$N_{E-F} = 0$$



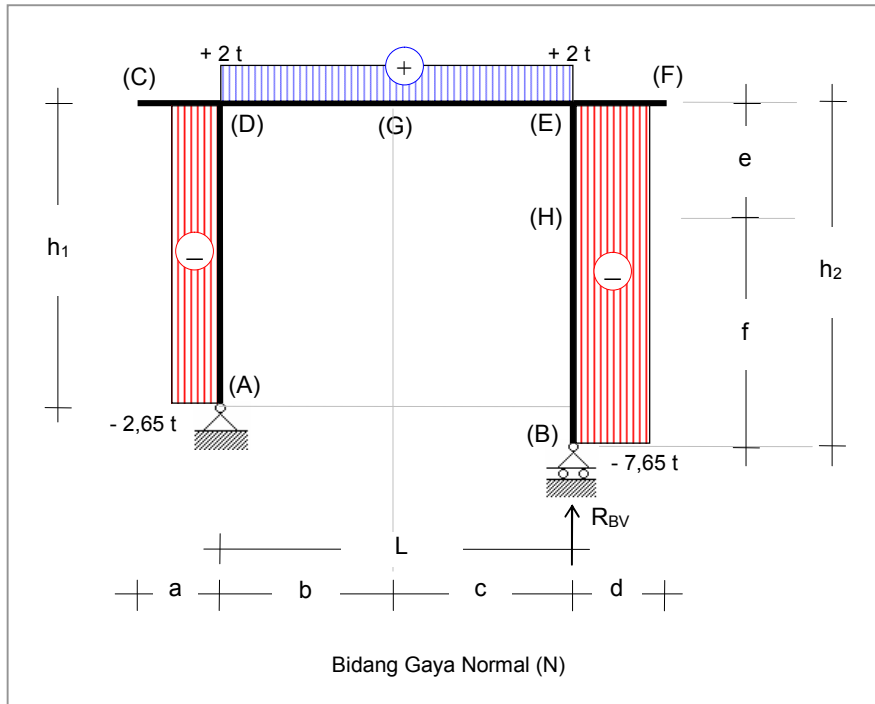
Gambar 17 : Putaran momen pada titik D dan titik E.



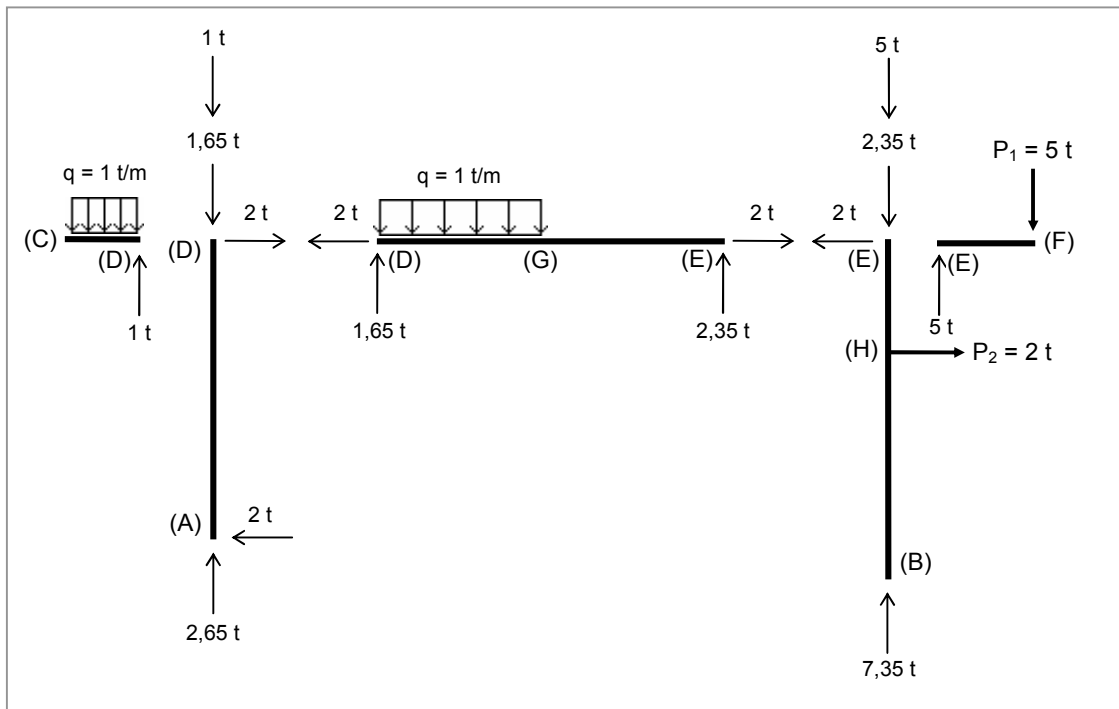
Gambar 18.a : Bidang gaya lintang.



Gambar 18.b : Bidang momen.

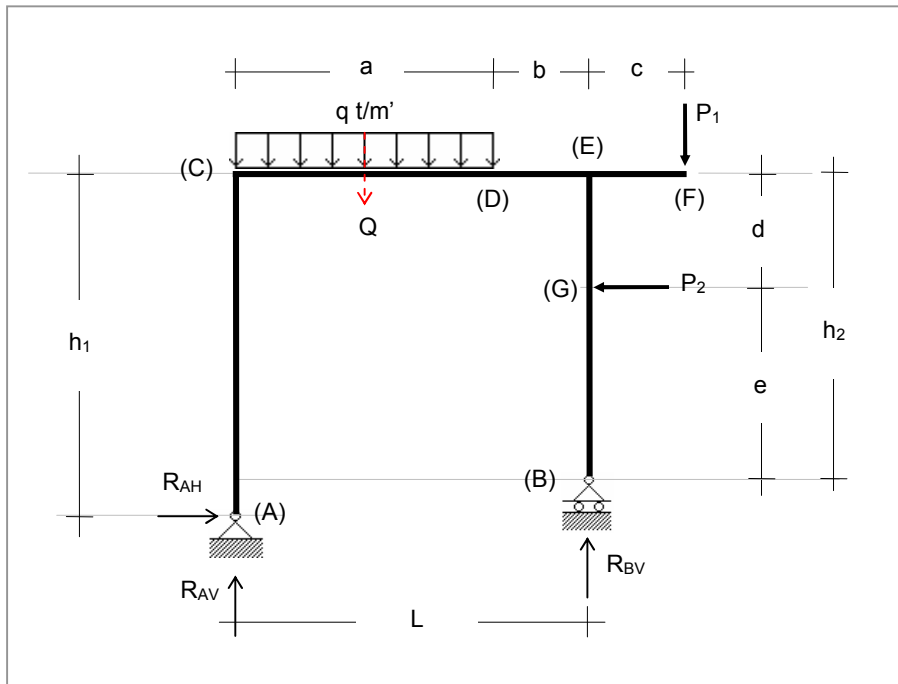


Gambar 18.c : Bidang gaya normal.



Gambar 19 : Distribusi gaya-gaya pada elemen struktur (balok/kolom) dengan cara freebody.

WORKSHOP/PELATIHAN



Diketahui : Struktur portal seperti tergambar.

Data-data,

PANJANG DAN TINGGI

L	a	b	c	d	e	h ₁	h ₂
m	m	m	m	m	m	m	m
12	10	2	2	3	4	8	7

BEBAN KERJA

No. Stb.	q t/m'	Q ton	P ₁ ton	P ₂ ton	Q + P ₁ ton
-1	1.0	10.0	2.0	2.0	12.000
0	1.2	12.0	2.2	2.1	14.200
1	1.4	14.0	2.4	2.2	16.400
2	1.6	16.0	2.6	2.3	18.600
3	1.8	18.0	2.8	2.4	20.800
4	2.0	20.0	3.0	2.5	23.000
5	2.2	22.0	3.2	2.6	25.200
6	2.4	24.0	3.4	2.7	27.400
7	2.6	26.0	3.6	2.8	29.600
8	2.8	28.0	3.8	2.9	31.800
9	3.0	30.0	4.0	3.0	34.000

Diminta : Gambarkan bidang-bidang gaya lintang, momen dan gaya normal pada seluruh bentang.

Penyelesaian :

a. Reaksi Perletakan.

$$Q = q \cdot a = (1 \text{ t/m}') \cdot (10 \text{ m}) = 10 \text{ ton.}$$

$$\Sigma H = 0,$$

$$R_{AH} - P_2 = 0$$

$$R_{AH} = P_2 = 2 \text{ ton (ke kanan).}$$

$$\begin{aligned}\Sigma M_B &= 0, \\ R_{AV} \cdot L - R_{AH} \cdot (h_1 - h_2) - Q \cdot (1/2a + b) + P_1 \cdot c - P_2 \cdot e &= 0 \\ R_{AV} &= R_{AH} \cdot (h_1 - h_2)/L + Q \cdot (1/2a + b)/L - P_1 \cdot c/L + P_2 \cdot e/L \\ R_{AV} &= 2 \cdot (8 - 7)/12 + 10 \cdot (1/2 \cdot 10 + 2)/12 - 2 \cdot 2/12 + 2 \cdot 4/12 \\ R_{AV} &= 0,167 + 5,833 - 0,333 + 0,667 \\ R_{AV} &= 6,333 \text{ ton (ke atas)}.\end{aligned}$$

$$\begin{aligned}\Sigma M_A &= 0, \\ -R_{BV} \cdot L - P_2 \cdot (h_1 - d) + P_1 \cdot (L + c) + Q \cdot (1/2a) &= 0 \\ R_{BV} &= -P_2 \cdot (h_1 - d)/L + P_1 \cdot (L + c)/L + Q \cdot (1/2a)/L \\ R_{BV} &= -2 \cdot (8 - 3)/12 + 2 \cdot (12 + 2)/12 + 10 \cdot (1/2 \cdot 10)/12 \\ R_{BV} &= -0,833 + 2,333 + 4,167 \\ R_{BV} &= 5,667 \text{ ton (ke atas)}.\end{aligned}$$

Kontrol :

$$\begin{aligned}\Sigma V &= 0, \\ R_{AV} + R_{BV} &= Q + P_1 \\ 6,333 \text{ t} + 5,667 \text{ t} &= 10 \text{ t} + 2 \text{ t} \\ 12 \text{ ton} &= 12 \text{ ton (memenuhi)}.\end{aligned}$$

b. Gaya lintang.

$$\begin{aligned}D_{A-C} &= -R_{AH} = -2 \text{ ton.} \\ D_{CD} &= R_{AV} = +6,333 \text{ ton} \\ D_{D-E} &= R_{AV} - Q = 6,333 - 10 = -3,667 \text{ ton.} \\ D_{E-F} &= R_{AV} - Q + R_{BV} = +6,333 - 10 + 5,667 = +2 \text{ ton} = P_1. \\ D_{E-G} &= +R_{AH} = +2 \text{ ton}\end{aligned}$$

c. Momen .

$$\begin{aligned}M_A &= 0 \\ M_{CA} &= -R_{AH} \cdot h_1 = -2 \cdot 8 = -16 \text{ t.m}' \\ M_C &= -R_{AH} \cdot h_1 = -2 \cdot 8 = -16 \text{ t.m}' \\ M_D &= -R_{AH} \cdot h_1 + R_{AV} \cdot a - Q \cdot 1/2a = -2 \cdot 8 + 6,333 \cdot 10 - 10 \cdot 1/2 \cdot 10 \\ &= -16 + 63,33 - 50 \\ M_D &= -2,67 \text{ t.m}' \\ M_{ED} &= -R_{AH} \cdot h_1 + R_{AV} \cdot L - Q \cdot (1/2a + b) \\ &= -2 \cdot 8 + 6,333 \cdot 12 - 10 \cdot (1/2 \cdot 10 + 2) \\ &= -16 + 76 - 70 \\ M_{ED} &= -10,0 \text{ t.m}' \\ M_{EG} &= -P_2 \cdot d = -2 \cdot 3 = -6 \text{ t.m}' \\ M_{EF} &= -P_1 \cdot c = -2 \cdot 2 = -4 \text{ t.m}' \\ \text{Atau,} \\ M_{ED} &= M_{EG} + M_{EF} = -6 + (-4) = -10 \text{ t.m}'.\end{aligned}$$

Letak titik dimana terdapat momen maksimum positif pada C-D,

$$M_x = R_{AV} \cdot x - R_{AH} \cdot h_1 - 1/2 q \cdot x^2$$

$$d(M_x)/dx = D_x = 0,$$

$$R_{AV} - q \cdot x = 0$$

$$x = R_{AV} / q = 6,333/1 = 6,333 \text{ m (dari titik C)}.$$

$$\begin{aligned}M_{maks} &= 6,333 \cdot (6,333 \text{ m}) - 2 \cdot (8 \text{ m}) - 1/2 \cdot (1 \text{ t/m}') \cdot (6,333 \text{ m})^2 \\ &= +4,053 \text{ t.m}'.\end{aligned}$$

Letak titik dimana momen sama dengan nol pada bentang C-E,

$$M_x = R_{AV} \cdot x - R_{AH} \cdot h_1 - 1/2 q \cdot x^2 = 0$$

$$x^2 - (R_{AV}/1/2q) \cdot x + R_{AH} \cdot h_1/1/2q = 0$$

$$x^2 - (6,333/0,5) \cdot x + 2 \cdot 8/0,5 = 0$$

$$x^2 - 12,667 \cdot x + 32 = 0$$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{12,667 \pm \sqrt{(-12,667)^2 - 4.(1).(32)}}{2.(1)}$$

$$x_{1,2} = 6,333 \pm 2,848$$

$$x_1 = 6,333 + 2,848 = 9,181 \text{ m (dari C).}$$

$$x_2 = 6,333 - 2,848 = 3,485 \text{ m (dari C).}$$

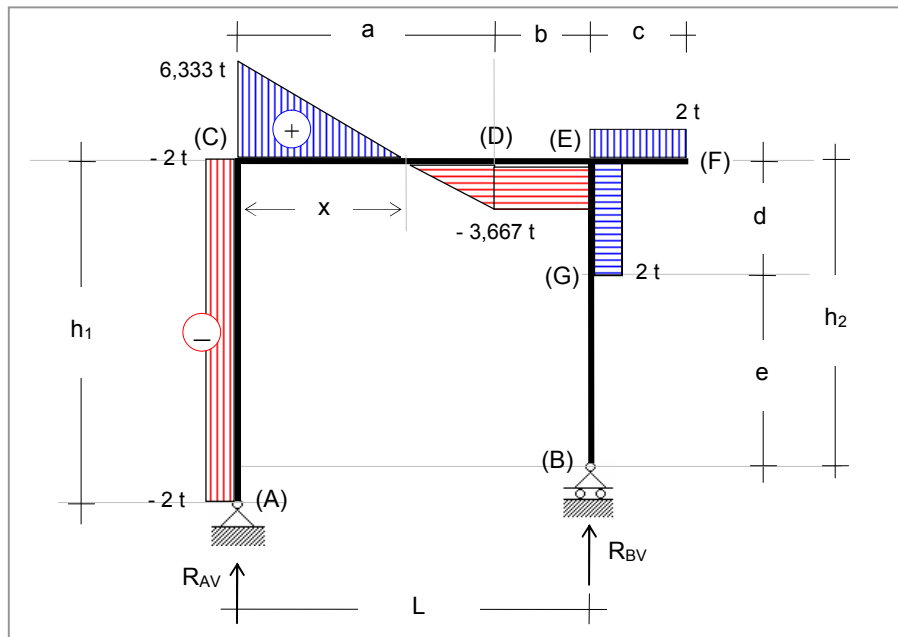
d. Gaya normal.

$$N_{A-C} = -R_{AV} = -6,333 \text{ ton (tekan).}$$

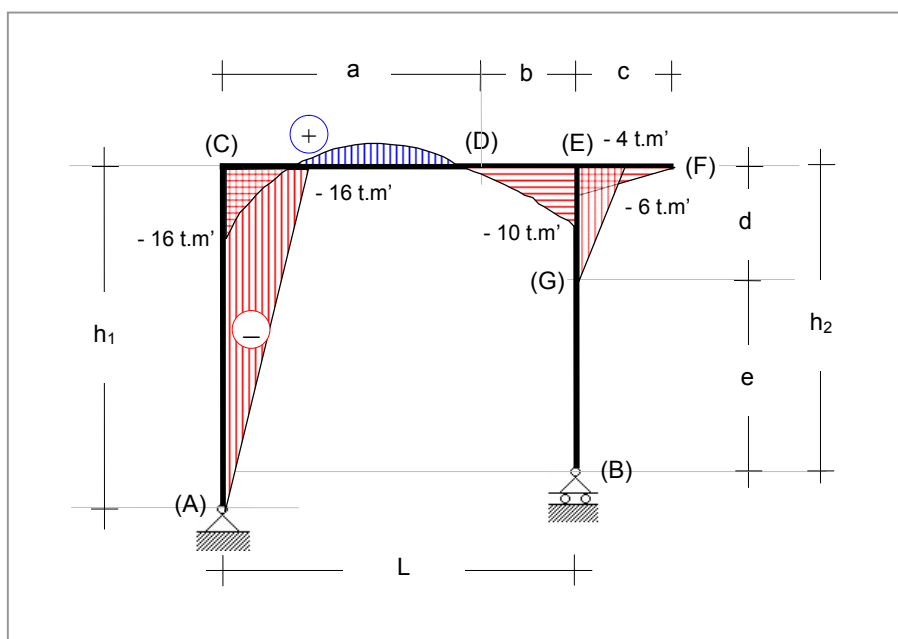
$$N_{C-E} = -R_{AH} = -2 \text{ ton (tekan).}$$

$$N_{E-B} = -R_{BV} = -5,667 \text{ ton (tekan).}$$

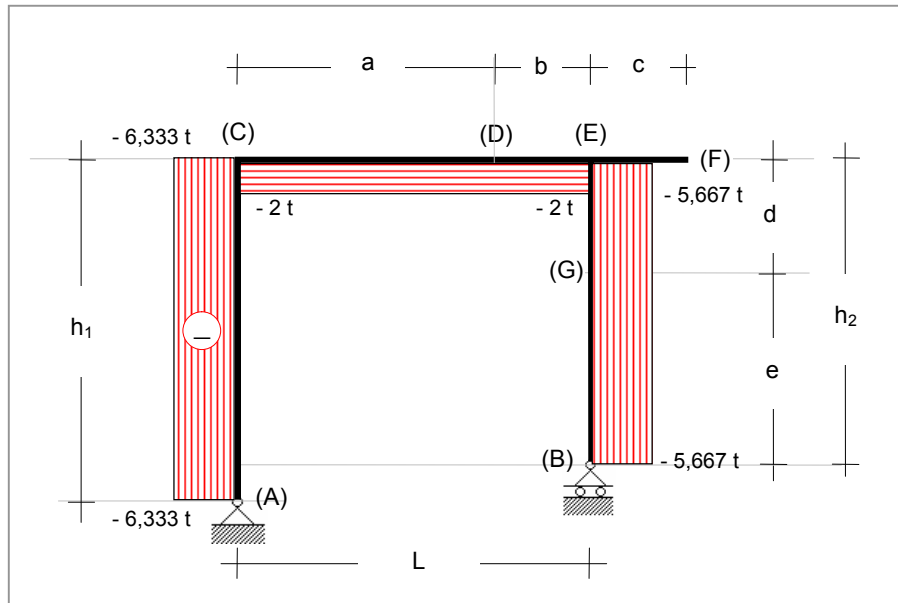
e. Gambar Bidang Gaya Lintang.



f. Gambar Bidang Momen.



g. Gambar Bidang Gaya Normal.



KUNCI JAWABAN

REAKSI PERLETAKAN

No. Stb.	R_{AH} ton	R_{AV} ton	R_{BV} ton	$R_{AV} + R_{BV}$ ton
-1	2.0	6.333	5.667	12.000
0	2.1	7.508	6.692	14.200
1	2.2	8.683	7.717	16.400
2	2.3	9.858	8.742	18.600
3	2.4	11.033	9.767	20.800
4	2.5	12.208	10.792	23.000
5	2.6	13.383	11.817	25.200
6	2.7	14.558	12.842	27.400
7	2.8	15.733	13.867	29.600
8	2.9	16.908	14.892	31.800
9	3.0	18.083	15.917	34.000

GAYA LINTANG

No. Stb.	D_{A-C} ton	D_{CD} ton	D_{D-E} ton	D_{E-F} ton	D_{E-G} ton
-1	-2.000	6.333	-3.667	2.000	2.000
0	-2.100	7.508	-4.492	2.200	2.100
1	-2.200	8.683	-5.317	2.400	2.200
2	-2.300	9.858	-6.142	2.600	2.300
3	-2.400	11.033	-6.967	2.800	2.400
4	-2.500	12.208	-7.792	3.000	2.500
5	-2.600	13.383	-8.617	3.200	2.600
6	-2.700	14.558	-9.442	3.400	2.700
7	-2.800	15.733	-10.267	3.600	2.800
8	-2.900	16.908	-11.092	3.800	2.900
9	-3.000	18.083	-11.917	4.000	3.000

MOMEN

No. Stb.	M_{CA} t.m'	M_D t.m'	M_{ED} t.m'	M_{EG} t.m'	M_{EF} t.m'	$M_{EG}+M_{EF}$ t.m'	x m	M_{maks} t.m'
-1	-16.000	-2.667	-10.000	-6.000	-4.000	-10.000	6.333	4.056
0	-16.800	-1.717	-10.700	-6.300	-4.400	-10.700	6.257	6.690
1	-17.600	-0.767	-11.400	-6.600	-4.800	-11.400	6.202	9.329
2	-18.400	0.183	-12.100	-6.900	-5.200	-12.100	6.161	11.971
3	-19.200	1.133	-12.800	-7.200	-5.600	-12.800	6.130	14.615
4	-20.000	2.083	-13.500	-7.500	-6.000	-13.500	6.104	17.261
5	-20.800	3.033	-14.200	-7.800	-6.400	-14.200	6.083	19.908
6	-21.600	3.983	-14.900	-8.100	-6.800	-14.900	6.066	22.555
7	-22.400	4.933	-15.600	-8.400	-7.200	-15.600	6.051	25.203
8	-23.200	5.883	-16.300	-8.700	-7.600	-16.300	6.039	27.852
9	-24.000	6.833	-17.000	-9.000	-8.000	-17.000	6.028	30.501

GAYA NORMAL

No. Stb.	N_{A-D} ton	N_{C-E} ton	N_{E-B} ton
-1	-6.333	-2.000	-5.667
0	-7.508	-2.100	-6.692
1	-8.683	-2.200	-7.717
2	-9.858	-2.300	-8.742
3	-11.033	-2.400	-9.767
4	-12.208	-2.500	-10.792
5	-13.383	-2.600	-11.817
6	-14.558	-2.700	-12.842
7	-15.733	-2.800	-13.867
8	-16.908	-2.900	-14.892
9	-18.083	-3.000	-15.917